Group Members: Charlie Sledz, Lohit Bhatnagar, Jackson Emerson, McKenna Marlar

Lab Division: 38

Lab Date: April 17th

Instructor: Avinash Bairwa

ME315 Heat and Mass Transfer

Final Report

Optimal Fin Design with Volume Constraint

Executive Summary

This project investigated the effect of fin geometry–rectangular, cylindrical, and conical–on heat dissipation under forced convection within a constrained volume of 2 inches in height, 1 inch in width, and 1 inch in depth. Fins are crucial in applications such as electronics, automotive systems, and HVAC equipment, where maximizing heat transfer in limited spaces is essential for performance and energy efficiency. The objective was to determine which fin design provides the greatest heat dissipation efficiency while confined to the specified volume.

Aluminum 6061 was selected for all fins due to its favorable thermal properties and widespread industrial use. Each fin, including its base plate, was machined as a single solid piece to minimize thermal contact resistance. Matte black paint was applied to enhance surface emissivity for accurate infrared (IR) imaging. Thermocouples were embedded into the base and attached to the tip of each fin to measure temperature during testing. All setups were placed on an electric heater embedded in insulation and subjected to 5 m/s airflow. Temperature data was collected continuously using LabVIEW until steady-state conditions were reached.

Preliminary theoretical calculations were conducted to estimate convective heat transfer coefficients, fin efficiencies, and heat dissipation rates based on standard correlations for forced convection. Assumptions included negligible radiation losses and nearly uniform temperature distributions along the fins, both supported by the experimental results.

Experimental data revealed key differences among the fin geometries. The rectangular fin exhibited the highest volumetric efficiency at 0.133 W/cm^3, maximizing heat transfer within the given space. Meanwhile, the conical fin achieved the highest mass efficiency at 0.142 W/g, making it particularly suited for weight-sensitive applications like aerospace systems. The cylindrical fin proved to be mediocre, as it did not excel in either volumetric or mass efficiency.

Sources of error included imperfect thermocouple attachment using thermal tape, minor heat loss through insulation material, slight discrepancies between IR and thermocouple readings, and variations in surface finish. Despite these factors, the results remained consistent with theoretical expectations. For future work, improvements such as embedding thermocouples directly into the fin tips, utilizing higher-quality insulation, and refining the surface finish are recommended to enhance measurement accuracy.

Introduction

Heat sinks are used for quick and efficient heat dissipation from a heat source to ensure proper cooling and avoid critically high temperatures that could damage components (1). Heat sinks work by absorbing heat, from the hot surface, into the fins and dissipating that heat to the surrounding fluid, which is typically air. Fins with greater surface areas tend to achieve this process quicker since there is more contact between the fin and cooling fluid (2).

Heat sinks are used in a variety of applications, such as engine cooling in the automotive industry and computer cooling in the electronic industry. In both of these instances, the size of the heat sink is spatially limited. For computers, technological advancements have led to smaller and slimmer designs, meaning there is less room for the various components needed to make a computer run. Thus, the heat sink needs to have the greatest efficiency in the smallest volume to allow for the best performance. Fin efficiency can be altered based on three factors: fin geometry, quantity, and material. For fin arrays, the spacing between fins and their orientation also impact heat sink efficiency. However, this experiment focused solely on fin geometry and its relationship with heat dissipation and fin efficiency.

To observe the solitary effects of changing fin geometry, a few control variables were defined. Firstly, to simplify the machining process, plates with only one fin were tested in each trial. Although typical heat sinks include numerous fins in an array, it can be reasoned that comparing a single fin to a single fin would yield approximately the same result as comparing two arrays with the same fin quantities. Secondly, a volume constraint was created to ensure that each design could fit into the same theoretical space within the heat sink. The chosen volume boundary was two inches in height, one inch in width, and one inch in depth. The base plate of each fin was also confined to having a nine-inch surface area with a quarter inch thickness. Lastly, each fin was machined using the same aluminum 6061 bar for material similarity. Both aluminum and copper are commonly used metals for heat sinks. Copper has a much better thermal conductivity than aluminum, but it is expensive and dense, making it less desirable for certain applications (3). Thus, the material selection for this experiment is similar to what would be used in industry and can be more accurately compared to real-world scenarios.

Rectangular, conical, and cylindrical fins were created for this analysis. These designs were chosen based on the current geometries used in industry and ease of manufacturing. The overarching goal was to determine the heat transfer rate and fin efficiency of each fin design, then to compare these values to determine which geometry would result in the greatest performance within the volume constraint. Using preliminary calculations, it was hypothesized that the cylindrical model would be the most efficient design based on volume, whereas the conical model would be the most efficient design based on mass.

Experimental Method

The objective of this experiment was to determine which of three different fin geometries provides the best performance in terms of volume efficiency and mass efficiency for dissipating heat. These fin geometries were rectangular, cylindrical, and conical. These metrics were calculated using temperature data collected from the base plate, fin base, and tip of each fin during testing.

Figure 1 and **Figure 2** below illustrate three fin designs; all machined from aluminum 6061 stock as a single solid piece including both the fin and the base plate. This eliminates the need for thermal paste or mechanical bonding and ensures uniform thermal conductivity throughout the object and no contact resistance considerations.

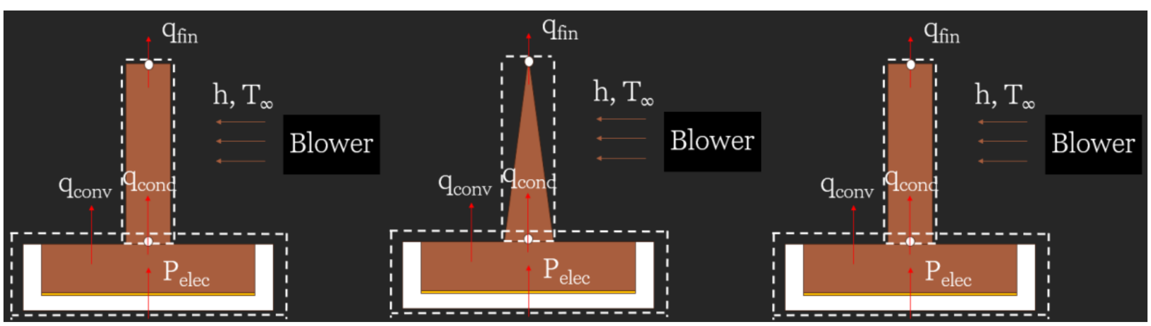


Figure 1. Side View of Experimental Setup (left to right): rectangular fin, conical fin, and cylindrical fin. White circles represent thermocouple placement. Fin boundary box is 1 in by 2 in.

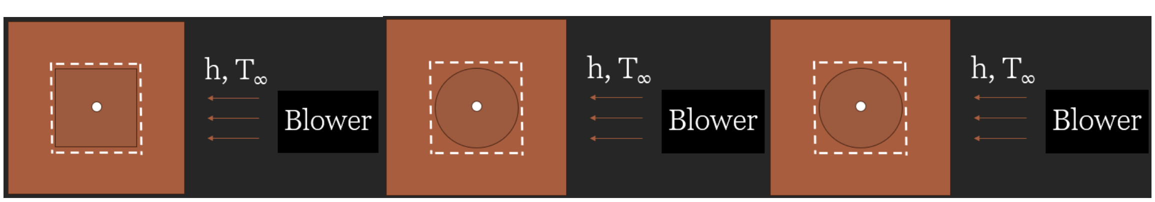


Figure 2. Top View of Experimental Setup (left to right): rectangular fin, conical fin, and cylindrical fin. White circles represent thermocouple placement. Fin boundary box is 1 in by 1 in.

To create a fair comparison, all fins were designed to fit within a strict bounding volume of 2 inches tall, 1 inch wide, and 1 inch deep. The base plate dimensions were 3 inches by 3 inches with a thickness of 0.25 inches. In addition to the finned samples, a flat plate with no fin was also manufactured from the same aluminum stock. This configuration served as a baseline case to calculate the heat transfer coefficient *h.* The flat plate allowed for direct comparison and provided a reference point to evaluate fin effectiveness.

After machining, each fin model was painted matte black to enhance emissivity for accurate temperature readings using an infrared (IR) camera. For direct thermal measurements, small holes were drilled into the base of each fin to insert thermocouples. To measure tip temperature, an additional thermocouple was taped to the tip for each fin on the opposite side of the airflow to avoid interference with heat transfer.

The samples were positioned on an electrical heater embedded in a polystyrene insulation block. The block was placed 2 inches in front of a blower box delivering a constant airflow of 5 m/s, determined by using an anemometer. The heater was connected to a Powerstat and a wattmeter and operated at around 28.8 Watts, accounting for any system power losses.

Once the system was fully assembled and powered, LabVIEW was used to collect thermal data until steady-state conditions were achieved. Afterward, images were taken, using an IR camera, from the top and side to determine the base temperature. Finally, the system was powered down and allowed to cool to room temperature before repeating the procedure for the next sample.

Analysis

Before the experiment was performed, detailed analysis of the setup was conducted to determine the theoretical results so that they could be compared with the experimental results. Firstly, the convective heat transfer coefficient was determined (see **Appendix B**). This was done by determining the Nusselt number for each case via an empirical correlation based on the Reynolds number. From the Nusselt number, the convective heat transfer coefficient, *h*, could then be determined. In addition to these values above, the fin efficiency, η_{fin}, for each fin was also of interest. These values were also determined from the fin's geometry and each case's m parameter. Using the appropriate properties of the air and the geometry of the fins, the results of each case can be found in **Table 1** below.

Table 1. Convective Heat Transfer Correlations for Fin Geometries

Configuration	Re_{L_c} [-]	Nu_{L_c} [-]	$h\ [\frac{W}{m^2K}]$	$m\ [m^{-1}]$	η_{fin} [-]
Flat Plate	22800	89.3	31.6	-	-
Rectangular Fin	19100	93.9	39.9	3.64	.983
Cylindrical Fin	17200	71.0	33.6	4.72	.976
Conical Fin	7640	43.0	45.7	5.51	.997

Based on these results, the team predicted the conical fin would have the highest convective heat transfer coefficient and be the most efficient, while the cylindrical fin would have the lowest convective heat transfer coefficient and be the least efficient.

Further analysis of the system was performed, based on the assumptions that the temperature gradients of each fin would be identical and that the excess base temperature, θ_b, would be equal to 35 K. While this assumption is not perfectly valid, it was made to assist in predicting the results of the experiment. Additionally, the experimental excess base temperature for each trial lay between 29 K and 39 K, proving the assumption of 35 K to be reasonable. In **Table 2** below, the predicted fin heat rate, q_{fin}, fin effectiveness, ϵ_{fin}, volumetric efficiency, η_V, and mass efficiency, η_m, can be seen for each fin geometry.

Table 2. Theoretical Heat Transfer and Efficiencies for Fin Geometries

Configuration	$q_{fin}\ [W]$	ϵ_{fin} [-]	$\eta_V\ [\frac{W}{cm^3}]$	$\eta_m\ [\frac{W}{g}]$
Rectangular Fin	4.43	4.91	0.135	0.0500
Cylindrical Fin	5.23	8.79	0.160	0.0752
Conical Fin	3.33	4.11	0.102	0.144

Based on these results, it was predicted that the cylindrical fin would have the highest heat rate, fin effectiveness, and volumetric efficiency, while the conical fin would have the highest mass efficiency. These theoretical results give approximate calculations, but they do not consider the actual temperature gradient present in each fin due to the previously discussed assumption. This is important to note because the experimental temperature gradients were used in the final results.

Results & Discussion

While the fins were heated, both the base and tip of the fin increased in temperature. This increase in temperature decreased as the system approached steady state. The data and related figures representing this can be seen in **Appendix D**. Notably, the systems never reached true steady state, as that would not be practical; however, each trial continued until the rate of change of the temperature of the base over time reached a sufficiently low value that could be used to approximate steady state. For the 8 trials performed, the range of this rate varied from 0.0892 $\frac{K}{s}$ to 0.135 $\frac{K}{s}$.

In **Table 3** below, the final excess temperatures at the base, θ_b, and tip, θ_L, of each trial and configuration can be seen. These final excess temperatures ranged from 29.76 K to 38.97 K and 16.77 K to 26.46 K for the base and tip, respectively. The excess temperatures were all based on the temperature of the surroundings at the time of the experiment, which was 297.6 K.

Table 3. Experimental Excess Temperatures of System Configurations

Configuration	Trial	θ_b [K]	θ_L [K]
Flat Plate	1	37.70	-
	2	38.97	-
Rectangular Fin	1	33.77	19.66
	2	30.73	19.40
Cylindrical Fin	1	32.15	24.27
	2	29.76	16.77
Conical Fin	1	38.81	26.46
	2	36.41	24.65

From the excess temperatures above and the known power input, the total thermal resistance, R_{total}, of the system to the surroundings can be determined. For the various system configurations, the total resistances varied from 1.05 $\frac{K}{W}$ to 1.37 $\frac{K}{W}$. Furthermore, the total resistance and total area in contact with the air flow can be used to determine the total convective heat transfer coefficient of the system. For the various system configurations, the total convective heat transfer coefficients varied from 92.1 $\frac{K}{W}$ to 129.7 $\frac{K}{W}$. The total resistance and total convective heat transfer coefficient for each configuration and trial can be seen in **Table 4** below, while the relations used to calculate these values and the calculations for each trial can be seen in **Appendix C**.

Table 4. Total Thermal Resistance & Total Convective Heat Transfer Coefficients of System Configurations

Configuration	Trial	R_{total} $[\frac{K}{W}]$	h_{total} $[\frac{W}{m^2K}]$
Flat Plate	1	1.33	129.7
	2	1.37	125.5
Rectangular Fin	1	1.16	103.1
	2	1.08	110.1
Cylindrical Fin	1	1.10	92.1
	2	1.05	96.8
Conical Fin	1	1.37	99.0
	2	1.27	106.4

While the above results are interesting to note, they are not particularly useful in real world applications, as no well-designed heat dissipation device using fins would have a large flat plate around its base. In practice, fins would be arranged in a grid to greatly increase the heat dissipation of the system. These results are important, though, to support the assumption that the heat transfer due to radiation in the system is negligible. The radiative heat transfer coefficients for each configuration and trial for this experiment ranged from 0.0103 to 0.0107 (see **Appendix C**), which are all much smaller than the above convective heat transfer coefficients. This means that radiation accounted for less than 1% of the heat transfer out of the system after it had reached steady state.

The primary results of these experiments are the various fin parameters, which were determined through a computerized iterative process that altered the fin convective heat transfer coefficient until the ratio of the tip excess temperature to the base excess temperature, $\frac{\theta_L}{\theta_B}$, matched the experiment. This was done using Microsoft Excel's "goal seek" function, and the equations used in this process can be seen in **Appendix C**. Those parameters involved in the iterative process, which include the fin convective heat transfer coefficient, h_{fin},the fin parameter, m, and the excess temperature ratio, $\frac{\theta_L}{\theta_b}$, are summarized in **Table** 5 below.

Table 5. Experimental Fin Parameters for Various Geometries

Configuration	Trial	$\frac{\theta_L}{\theta_b}$ [-]	h_{fin} [$\frac{W}{m^2K}$]	m [m^{-1}]
Rectangular Fin	1	0.582	25.45	16.40
	2	0.631	21.06	14.92
Cylindrical Fin	1	0.755	15.00	12.59
	2	0.564	33.20	18.73
Conical Fin	1	0.682	85.94	30.14
	2	0.682	85.94	30.14

In addition to the previously determined fin parameters, the fin heat rate, q_{fin}, and fin efficiency, were also determined using the above parameters and the geometry of each fin (see **Appendix C**). These values, along with their associated percent error, can be seen below in **Table 6**.

Table 6. Experimental Fin Heat Rate & Efficiency for Various Geometries

Configuration	Trial	q_{fin} [W]	Percent Error	n_f [-]	Percent Error
Rectangular Fin	1	4.35	1.84%	0.75	23.9%
	2	3.43	22.47%	0.78	20.7%
Cylindrical Fin	1	2.03	61.2%	0.86	12.2%

	2	3.50	33.2%	0.74	24.5%
Conical Fin	1	3.28	1.46%	0.91	8.23%
	2	3.06	8.24%	0.91	8.23%

Finally, the fin effectiveness and efficiencies of interest, the volumetric efficiency and mass efficiency could be determined from the fin heat rates and the fins geometry (see **Appendix C**). These values and their associated percent errors can be seen in **Table 7** below.

Table 7. Fin Effectiveness and Volumetric & Mass Efficiencies for Various Geometries

Configuration	Trial	$\in$ [-]	Error	η_V [-]	η_m [-]	Error
Rectangular Fin	1	7.84	59.6%	0.133	0.049	1.85%
	2	8.22	67.3%	0.105	0.039	22.5%
Cylindrical Fin	1	8.30	5.51%	0.062	0.029	61.2%
	2	6.98	20.5%	0.107	0.050	33.1%
Conical Fin	1	1.94	52.7%	0.100	0.142	1.46%
	2	1.94	52.7%	0.093	0.132	8.24%

The mass and volumetric efficiencies share the same percent error due to their direct relationship to the fin heat rate, so they have been joined together in the final column of **Table 7**. Notably, the conical fin geometry is the only fin to have a fin effectiveness of less than 2, meaning its use in heat transfer applications is usually not justifiable, while the remaining fin geometries are easily justifiable, with fin effectivenesses that are all well above 2. Additionally, the fin efficiencies, with the exception of the cylindrical fin, are very similar to the expected values, supporting the preliminary analysis and the results of the experiment. Lastly, the conical fin has the highest mass efficiency, and contrary to the expectation, the rectangular fin has the highest volumetric efficiency.

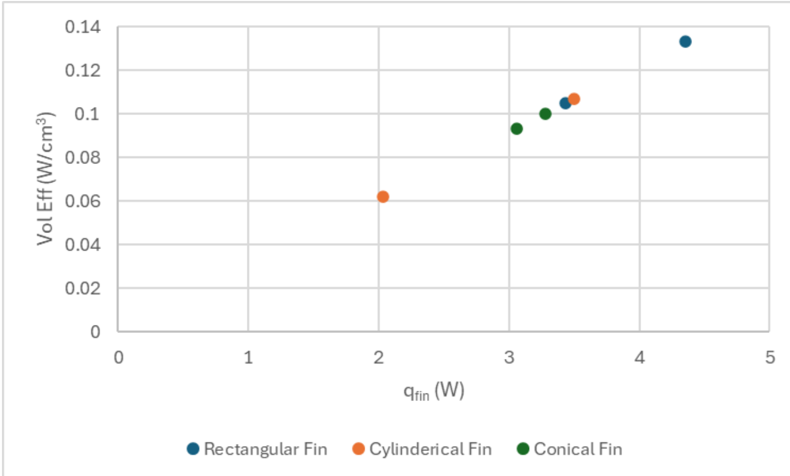


Figure 3. Volumetric Efficiency vs. Fin Heat Rate of the 3 fin designs

Figure 3 shows the relationship between volumetric efficiency and the fin heat rate for the rectangular, cylindrical, and conical fin designs. The rectangular fin consistently demonstrates the highest volumetric efficiencies, reaching up to 0.133 W/cm^3. In comparison, the cylindrical and conical fins achieved lower volumetric efficiencies, with the cylindrical fin being the least efficient overall. The trend indicates that the rectangular fin, due to its ability to fully occupy the boundary box volume, provided superior heat dissipation per unit volume. Meanwhile, the conical and cylindrical fins showed similar but lower efficiencies, suggesting that geometry plays a significant role in optimizing space-constrained heat sink designs.

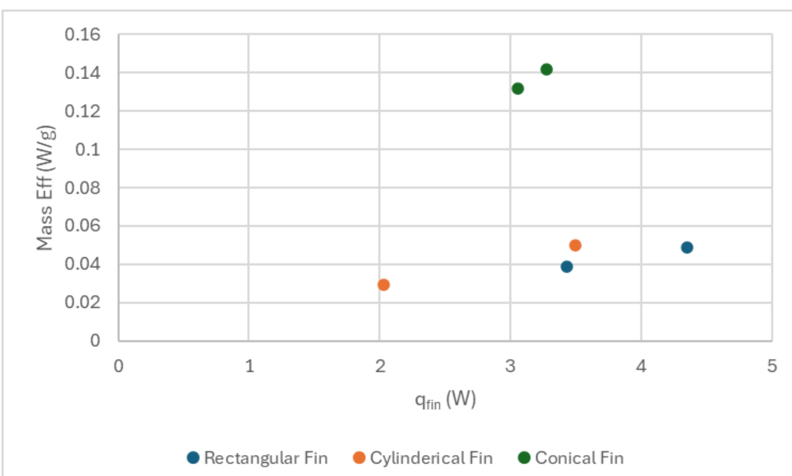


Figure 4. Mass Efficiency vs. Fin Heat Rate of the 3 fin designs

Figure 4 demonstrates the relationship between mass efficiency and the fin heat rate for the rectangular, cylindrical, and conical fins. There is little correlation between the various geometries, but as the fin heat rate increases, the mass efficiency does too, meaning that these could be more beneficial in high heat transfer applications. The conical fin consistently achieved the highest mass efficiencies, reaching 0.142 W/g. This trend highlights the advantage of the conical fin's reduced mass, allowing for greater heat dissipation per gram compared to the other designs, thus the conical fin could be preferred in applications like aerospace where minimizing weight is critical without sacrificing thermal performance.

While the experiment was successful overall and the expected trends were observed, there were some sources of error. The tip thermocouples were taped to the opposite side of the airflow instead of being embedded into the fin, which likely caused slight inaccuracies in temperature readings due to imperfect thermal contact. Part of the bottom insulation, used for the heating element, melted during testing. This potentially led to additional heat loss to the surroundings and an underestimation of the heat transfer rate. Differences between the IR camera readings and the thermocouple data, as well as small sections of chipped paint, also introduced uncertainty in surface temperature measurements. To improve the accuracy of future experiments, thermocouples could be embedded into the fins, more durable insulation materials should be used, and surface finishes could be smoother to ensure better thermal consistency.

Despite the sources of error, the results can guide future engineers in selecting fin geometries to maximize performance, whether it is minimizing weight in compact designs or working with limited space for a heat sink.

Conclusion

The aim of this experiment was to determine which fin geometry–rectangular, cylindrical, or conical–provides the best performance in terms of volume efficiency and mass efficiency for cooling applications.

It was hypothesized that the rectangular fin would dissipate the most heat within the boundary box due to maximizing surface area, and the cylindrical fin would dissipate the most heat per unit mass because it being lighter than rectangular fin but still having a large surface area.

The fins and plates were machined as single solid bodies from aluminum 6061, drilled to insert thermocouples at the base, and painted black for infrared measurements. Taped thermocouples were added to the fin tips. A flat plate was also manufactured to find the base heat transfer coefficient. Each setup was heated and tested under a 5 m/s airflow.

The major accomplishments achieved from this experiment are the following:

- Rectangular fin had the best volume efficiency at 0.105 W/m^3 and 0.133 W/m^3
- Conical fin had the best mass efficiency at 0.142 W/kg and 0.132 W/kg

Future work could explore using different materials, such as copper or composite alloys, to study the impact of thermal conductivity and density of fin performance. Testing a wider variety of fin geometries, like triangular or elliptical shapes, would also help identify more efficient designs. These extensions would provide deeper insight into optimizing heat sink performance for different engineering applications.

References

(1) "Heat Sink Design: Basics, Principle, and Practical Tips | AT-Machining." *Https://At-Machining.com/*, 19 Mar. 2019, at-machining.com/heat-sink-design/. Accessed 22 Apr. 2025.

(2) "What Is a Heatsink? | How It Works, Types & Manufacturing Process." *Radian*, 14 Apr. 2021, radianheatsinks.com/heatsink/. Accessed 22 Apr. 2025.

(3) Sinks, ABL Heat. "Material." *ABL*, www.abl-heatsinks.co.uk/technical/material. Accessed 22 Apr. 2025.

Appendix

Appendix A Theoretical analysis of equations, tables and derivations

A.1 Fin Heat Transfer Rate

TABLE 3.4 Temperature distribution and heat loss for fins of uniform cross section

Case	Tip Condition $(x = L)$	Temperature Distribution θ/θ_b		Fin Heat Transfer Rate q_f	
A	Convection heat transfer: $h\theta(L) = -kd\theta/dx\vert_{x=L}$	$\dfrac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$	(3.70)	$M\dfrac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$	(3.72)
B	Adiabatic $d\theta/dx\vert_{x=L} = 0$	$\dfrac{\cosh m(L-x)}{\cosh mL}$	(3.75)	$M \tanh mL$	(3.76)
C	Prescribed temperature: $\theta(L) = \theta_L$	$\dfrac{(\theta_L/\theta_b)\sinh mx + \sinh m(L-x)}{\sinh mL}$	(3.77)	$M\dfrac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$	(3.78)
D	Infinite fin $(L \to \infty)$: $\theta(L) = 0$	e^{-mx}	(3.79)	M	(3.80)

$\theta \equiv T - T_\infty$ $m^2 \equiv hP/kA_c$
$\theta_b = \theta(0) = T_b - T_\infty$ $M \equiv \sqrt{hPkA_c}\theta_b$

A.2 Pin Fin Efficiency

TABLE 3.5 Efficiency of common fin shapes

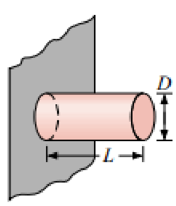
Pin Fins

Rectangular[b]

$A_f = \pi D L_c$

$L_c = L + (D/4)$

$V = (\pi D^2/4)L$



$$\eta_f = \frac{\tanh mL_c}{mL_c} \quad (3.95)$$

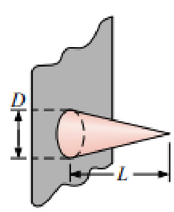
A.3 Conical Fin Efficiency

TABLE 3.5 Efficiency of common fin shapes

Triangular[b]

$A_f = \dfrac{\pi D}{2}[L^2 + (D/2)^2]^{1/2}$

$V = (\pi/12)D^2 L$



$$\eta_f = \frac{2}{mL}\frac{I_2(2mL)}{I_1(2mL)} \quad (3.96)$$

A.4 Nusselt Number Equation

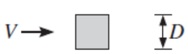
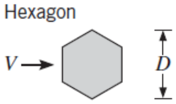
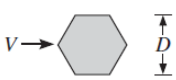
$$\overline{Nu}_D \equiv \frac{\bar{h}D}{k} = C\,Re_D^m\,Pr^{1/3} \tag{7.52}$$

A.5 Nusselt Number C and m values

TABLE 7.2 Constants of Equation 7.52 for the circular cylinder in cross flow [11, 12]

Re_D	C	m
0.4–4	0.989	0.330
4–40	0.911	0.385
40–4000	0.683	0.466
4000–40,000	0.193	0.618
40,000–400,000	0.027	0.805

TABLE 7.3 Constants of Equation 7.52 for noncircular cylinders in cross flow of a gas [14, 15][a]

Geometry		Re_D	C	m
Square				
		6000–60,000	0.304	0.59
		5000–60,000	0.158	0.66
Hexagon				
		5200–20,400	0.164	0.638
		20,400–105,000	0.039	0.78
		4500–90,700	0.150	0.638
Thin plate perpendicular to flow				
	Front	10,000–50,000	0.667	0.500
	Back	7000–80,000	0.191	0.667

[a]These tabular values are based on the recommendations of Sparrow et al. [15] for air, with extension to other fluids through the $Pr^{1/3}$ dependence of Equation 7.52. A Prandtl number of $Pr = 0.7$ was assumed for the experimental results for air that are described in [15].

A.6 Nusselt Number Formula for Flat Plate

TABLE 7.7 **Summary of convection heat transfer correlations for external flow**[a,b]

Correlation		Geometry	Conditions[c]
$\delta = 5x\, Re_x^{-1/2}$	(7.19)	Flat plate	Laminar, T_f
$C_{f,x} = 0.664\, Re_x^{-1/2}$	(7.20)	Flat plate	Laminar, local, T_f
$Nu_x = 0.332\, Re_x^{1/2}\, Pr^{1/3}$	(7.23)	Flat plate	Laminar, local, T_f, $Pr \gtrsim 0.6$
$\delta_t = \delta\, Pr^{-1/3}$	(7.24)	Flat plate	Laminar, T_f
$\overline{C}_{f,x} = 1.328\, Re_x^{-1/2}$	(7.29)	Flat plate	Laminar, average, T_f
$\overline{Nu}_x = 0.664\, Re_x^{1/2}\, Pr^{1/3}$	(7.30)	Flat plate	Laminar, average, T_f, $Pr \gtrsim 0.6$
$Nu_x = 0.564\, Pe_x^{1/2}$	(7.32)	Flat plate	Laminar, local, T_f, $Pr \lesssim 0.05$, $Pe_x \gtrsim 100$
$C_{f,x} = 0.0592\, Re_x^{-1/5}$	(7.34)	Flat plate	Turbulent, local, T_f, $Re_x \lesssim 10^8$
$\delta = 0.37x\, Re_x^{-1/5}$	(7.35)	Flat plate	Turbulent, T_f, $Re_x \lesssim 10^8$
$Nu_x = 0.0296\, Re_x^{4/5}\, Pr^{1/3}$	(7.36)	Flat plate	Turbulent, local, T_f, $Re_x \lesssim 10^8$, $0.6 \lesssim Pr \lesssim 60$
$\overline{C}_{f,L} = 0.074\, Re_L^{-1/5} - 1742\, Re_L^{-1}$	(7.40)	Flat plate	Mixed, average, T_f, $Re_{x,c} = 5 \times 10^5$, $Re_L \lesssim 10^8$
$\overline{Nu}_L = (0.037\, Re_L^{4/5} - 871)Pr^{1/3}$	(7.38)	Flat plate	Mixed, average, T_f, $Re_{x,c} = 5 \times 10^5$, $Re_L \lesssim 10^8$, $0.6 \lesssim Pr \lesssim 60$

Appendix B Full Theoretical Process for Each Fin

B.1 Flat Plate

Given information

$$u_{air} = 5\ m/s$$

$$v_{air} = 1.66 * 10^{-5}\ \frac{m^2}{s}$$

$$L_c = 0.0762\ m$$

$$k_{air} = 14.9\ \frac{w}{mK}$$

Reynolds Number

$$Re_L = \frac{u_{air} L_c}{v_{air}} = 22900$$

Nusselt Number

Using Table 7.7, Average, Laminar

$$Nu_L = 0.664 Re_L^{1/2} Pr_{air}^{1/3} = 89.3$$

Coefficient of Heat Transfer

$$h = \frac{Nu_L k_{air}}{L_c} = 31.6 \frac{w}{m^2 k}$$

B.2 Rectangular Fin

Given information

$$u_{air} = 5\ m/s$$

$$v_{air} = 1.66 * 10^{-5}\ \frac{m^2}{s}$$

$$L_c = 0.0635\ m$$

$$Pr = 0.7$$

$$k_{aluminum} = 237 \ \frac{w}{mK}$$

Reynolds Number

$$Re_L = \frac{u_{air} L_c}{v_{air}} = 19100$$

Nusselt Number

Using Table 7.3, C = 0.158, m = 0.66

$$Nu_L = 0.158 Re_L^{0.66} Pr_{air}^{1/3} = 93.9$$

Coefficient of Heat Transfer

$$h = \frac{Nu_L k_{air}}{L_c} = 39.9 \frac{w}{m^2 k}$$

Fin Efficiency

$$m = \sqrt{\frac{2h}{k_{aluminum} D}} = 3.64 \, m^{-1}$$

$$n_f = \frac{\tanh{(mL_c)}}{mL_c} = 0.983$$

B.3 Cylindrical Fin

Given information

$$u_{air} = 5 \, m/s$$

$$v_{air} = 1.66 * 10^{-5} \ \frac{m^2}{s}$$

$$L = 0.0826 \, m$$

$$D = 0.0254 \, m$$

$$L_c = L + \frac{D}{4} = \ 0.0572 \, m$$

$$Pr = 0.7$$

$$k_{aluminum} = 237\ \frac{w}{mK}$$

Reynolds Number

$$Re_D = \frac{u_{air} L_c}{v_{air}} = 17200$$

Nusselt Number

Using Table 7.2, C = 0.193, m = 0.618

$$Nu_D = 0.193 Re_L^{0.618} Pr_{air}^{1/3} = 71.0$$

Coefficient of Heat Transfer

$$h = \frac{Nu_D k_{air}}{D} = 33.6 \frac{w}{m^2 k}$$

Fin Efficiency

$$m = \sqrt{\frac{2h}{k_{aluminum} D}} = 3.64\ m^{-1}$$

$$n_f = \frac{\tanh\,(mL_c)}{mL_c} = 0.976$$

B.4 Cone Fin

Given information

$$u_{air} = 5\ m/s$$

$$v_{air} = 1.66 * 10^{-5}\ \frac{m^2}{s}$$

$$L = 0.0254\ m$$

$$D(y) = 0.0254y\ m$$

$$Pr = 0.7$$

$$k_{aluminum} = 237\ \frac{w}{mK}$$

Reynolds Number

$$Re_D = \int_0^L \frac{u_{air} D(y)}{v_{air}} = 7640$$

Nusselt Number

Using Table 7.2, C = 0.193, m = 0.618

$$Nu_D = 0.193 Re_D^{0.618} Pr_{air}^{1/3} = 71.0$$

Coefficient of Heat Transfer

$$h = \frac{Nu_D k_{air}}{D} = 45.74 \frac{w}{m^2 k}$$

Fin Efficiency

$$m = \sqrt{\frac{4h}{k_{aluminum} D}} = 5.51\, m^{-1}$$

$$n_f = \frac{2}{mL} \frac{I_2(2mL)}{I_1(2mL)} = 0.997$$

B.5 Radiation

$$h_r = \sigma(T_s^2 + T_{sur}^2)(T_s + T_{sur}) = 0.0106 \frac{w}{m^2 k}$$

$$h_r \ll h_{total}$$

Appendix C Full Experimental Process for Each Fin

C.1 Flat Plate

Given Information

$$T_{s,1} = 62.1\ °C$$

$$T_{s,2} = 63.37\ °C$$

$$T_{sur} = 24.4\ °C$$

$$P_{wattmeter} = 31.4\ W$$

$$P_{loss} = 3\ W$$

$$A = 0.00581\ m^2$$

Resistance

$$R_{total,1} = \frac{T_{s,1} - T_{sur}}{P_{wattmeter} - P_{loss}} = 1.33\frac{K}{W}$$

$$R_{total,2} = \frac{T_{s,2} - T_{sur}}{P_{wattmeter} - P_{loss}} = 1.37\frac{K}{W}$$

Convective Heat Transfer Coefficient

$$h_1 = \frac{1}{(R_{total,1}A)_{total}} = 129.74\frac{w}{m^2k}$$

$$h_2 = \frac{1}{(R_{total,2}A)_{total}} = 125.51\frac{w}{m^2k}$$

Radiative Heat Transfer Coefficient

$$h_{r,1} = \sigma(T_s^2 + T_{sur}^2)(T_s + T_{sur}) = 0.0106\frac{w}{m^2k}$$

$$h_{r,2} = \sigma(T_s^2 + T_{sur}^2)(T_s + T_{sur}) = 0.0107\frac{w}{m^2k}$$

C.2 Rectangular Fin

Given Information

$$T_{b,1} = 58.17\,°C$$

$$T_{b,2} = 55.13\,°C$$

$$T_{t,1} = 44.60\,°C$$

$$T_{t,2} = 43.80\,°C$$

$$T_{sur} = 24.4\,°C$$

$$P_{wattmeter,1} = 32.2\,W$$

$$P_{wattmeter,1} = 31.4\,W$$

$$P_{loss} = 3\,W$$

$$L_c = 0.0635\,m$$

$$A_t = 0.00839\,m^2$$

$$A_c = 0.00065\,m^2$$

$$P = 0.10\,m^2$$

$$k_{air} = 14.9\,\frac{w}{mK}$$

$$m_f = 0.0885\,kg$$

$$V_{box} = 3.278 * 10^{-5} m^3$$

$$\theta_{b,1} = T_b - T_{sur} = 33.77\,°C$$

$$\theta_{b,2} = T_b - T_{sur} = 30.73\,°C$$

Resistance

$$R_{total,1} = \frac{T_{b,1} - T_{sur}}{P_{wattmeter} - P_{loss}} = 1.16\,\frac{K}{W}$$

$$R_{total,2} = \frac{T_{b,2} - T_{sur}}{P_{wattmeter} - P_{loss}} = 1.08\,\frac{K}{W}$$

Total Coefficient of Heat Transfer

$$h_{total,1} = \frac{1}{R_{total,1}A_t} = 103.10\,\frac{w}{m^2k}$$

$$h_{total,2} = \frac{1}{R_{total,2}A_t} = 110.19\,\frac{w}{m^2k}$$

Radiative Heat Transfer Coefficient

$$h_{r,1} = \sigma(T_s^2 + T_{sur}^2)(T_s + T_{sur}) = 0.0105\,\frac{w}{m^2k}$$

$$h_{r,2} = \sigma(T_s^2 + T_{sur}^2)(T_s + T_{sur}) = 0.0103\,\frac{w}{m^2k}$$

Fin Coefficient of Heat Transfer

$$\frac{\theta_L}{\theta_{b,1}} = \frac{1}{\cosh(mL_c) + \frac{h}{mk_{air}}\sinh(mL_c)} \rightarrow h_{fin,1} = 25.45\,\frac{w}{m^2k}$$

$$\frac{\theta_L}{\theta_{b,2}} = \frac{1}{\cosh(mL_c) + \frac{h}{mk_{air}}\sinh(mL_c)} \rightarrow h_{fin,2} = 21.06\,\frac{w}{m^2k}$$

Fin parameter

$$m_1 = \sqrt{\frac{h_{fin,1}P}{k_{air}A_C}} = 16.40\,m^{-1}$$

$$m_2 = \sqrt{\frac{h_{fin,2}P}{k_{air}A_C}} = 14.92\,m^{-1}$$

Modified Heat Transfer Rate

$$M_1 = \theta_{b,1}(h_{fin.1}Pk_{air}A_c)^{1/2} = 5.32\,W$$

$$M_2 = \theta_{b,2}(h_{fin,2}Pk_{air}A_c)^{1/2} = 4.41\ W$$

Fin Heat Transfer

$$q_{fin,1} = M_1 \frac{\sinh(m_1L_c) + \frac{h_{fin.1}}{m_1k_{air}}\cosh(m_1L_c)}{\cosh(m_1L_c) + \frac{h_{fin.1}}{m_1k_{air}}\sinh(m_1L_c)} = 4.38\ W$$

$$q_{fin,2} = M_2 \frac{\sinh(m_2L_c) + \frac{h_{fin.2}}{m_2k_{air}}\cosh(m_2L_c)}{\cosh(m_2L_c) + \frac{h_{fin.2}}{m_2k_{air}}\sinh(m_2L_c)} = 3.43\ W$$

Fin Efficiency

$$\eta_{fin,1} = \frac{\tanh\ (m_1L_c)}{m_1L_c} = 0.75$$

$$\eta_{fin,2} = \frac{\tanh\ (m_2L_c)}{m_2L_c} = 0.78$$

Fin Effectiveness

$$\in_1 = \frac{q_{fin,1}}{\theta_{b,1}h_{fin,2}A_c} = 7.84$$

$$\in_2 = \frac{q_{fin,2}}{\theta_{b,2}h_{fin,2}A_c} = 8.22$$

Volume Efficiency

$$Volume\ Eff_1 = \frac{q_{fin,1}}{V_{box}} = 0.105\,\frac{w}{cm^3}$$

$$Volume\ Eff_2 = \frac{q_{fin,2}}{V_{box}} = 0.133\,\frac{w}{cm^3}$$

Mass Efficiency

$$Mass\ Eff_1 = \frac{q_{fin,1}}{m_f} = 0.049\,\frac{w}{g}$$

$$Mass\ Eff_2 = \frac{q_{fin,2}}{m_f} = 0.039\frac{w}{g}$$

C.3 Cylindrical Fin

Given Information

$$T_{b,1} = 56.55\ °C$$

$$T_{b,2} = 54.16\ °C$$

$$T_{t,1} = 48.67\ °C$$

$$T_{t,2} = 41.14\ °C$$

$$T_{sur} = 24.4\ °C$$

$$P_{wattmeter,1} = 32.2\ W$$

$$P_{wattmeter,1} = 31.4\ W$$

$$P_{loss} = 3\ W$$

$$L_c = 0.0572\ m$$

$$A_t = 0.00986\ m^2$$

$$A_c = 0.00051\ m^2$$

$$P = 0.108\ m^2$$

$$k_{air} = 14.9\ \frac{w}{mK}$$

$$m_f = 0.0695\ kg$$

$$V_{box} = 3.278 * 10^{-5} m^3$$

$$\theta_{b,1} = T_b - T_{sur} = 32.15\ °C$$

$$\theta_{b,2} = T_b - T_{sur} = 29.76\ °C$$

Resistance

$$R_{total,1} = \frac{T_{b,1} - T_{sur}}{P_{wattmeter} - P_{loss}} = 1.10\,\frac{K}{W}$$

$$R_{total,2} = \frac{T_{b,2} - T_{sur}}{P_{wattmeter} - P_{loss}} = 1.05\,\frac{K}{W}$$

Total Coefficient of Heat Transfer

$$h_{total,1} = \frac{1}{R_{total,1}A_t} = 92.11\,\frac{w}{m^2k}$$

$$h_{total,2} = \frac{1}{R_{total,2}A_t} = 96.78\,\frac{w}{m^2k}$$

Radiative Heat Transfer Coefficient

$$h_{r,1} = \sigma(T_s^2 + T_{sur}^2)(T_s + T_{sur}) = 0.0104\,\frac{w}{m^2k}$$

$$h_{r,2} = \sigma(T_s^2 + T_{sur}^2)(T_s + T_{sur}) = 0.0103\,\frac{w}{m^2k}$$

Fin Coefficient of Heat Transfer

$$\frac{\theta_L}{\theta_{b,1}} = \frac{1}{\cosh(mL_c) + \frac{h}{mk_{air}}\sinh(mL_c)} \rightarrow h_{fin,1} = 15.00\,\frac{w}{m^2k}$$

$$\frac{\theta_L}{\theta_{b,2}} = \frac{1}{\cosh(mL_c) + \frac{h}{mk_{air}}\sinh(mL_c)} \rightarrow h_{fin,2} = 32.20\,\frac{w}{m^2k}$$

Fin parameter

$$m_1 = \sqrt{\frac{h_{fin,1}P}{k_{air}A_C}} = 12.59\,m^{-1}$$

$$m_2 = \sqrt{\frac{h_{fin,2}P}{k_{air}A_C}} = 18.73\,m^{-1}$$

Modified Heat Transfer Rate

$$M_1 = \theta_{b,1}(h_{fin.1}Pk_{air}A_c)^{1/2} = 3.06\,W$$

$$M_2 = \theta_{b,2}(h_{fin,2}Pk_{air}A_c)^{1/2} = 4.21\,W$$

Fin Heat Transfer

$$q_{fin,1} = M_1 \frac{\sinh(m_1L_c) + \frac{h_{fin.1}}{m_1k_{air}}\cosh(m_1L_c)}{\cosh(m_1L_c) + \frac{h_{fin.1}}{m_1k_{air}}\sinh(m_1L_c)} = 2.03\,W$$

$$q_{fin,2} = M_2 \frac{\sinh(m_2L_c) + \frac{h_{fin.2}}{m_2k_{air}}\cosh(m_2L_c)}{\cosh(m_2L_c) + \frac{h_{fin.2}}{m_2k_{air}}\sinh(m_2L_c)} = 3.50\,W$$

Fin Efficiency

$$\eta_{fin,1} = \frac{\tanh\,(m_1L_c)}{m_1L_c} = 0.86$$

$$\eta_{fin,2} = \frac{\tanh\,(m_2L_c)}{m_2L_c} = 0.74$$

Fin Effectiveness

$$\epsilon_1 = \frac{q_{fin,1}}{\theta_{b,1}h_{fin,2}A_c} = 8.30$$

$$\epsilon_2 = \frac{q_{fin,2}}{\theta_{b,2}h_{fin,2}A_c} = 6.98$$

Volume Efficiency

$$Volume\ Eff_1 = \frac{q_{fin,1}}{V_{box}} = 0.062\,\frac{w}{cm^3}$$

$$Volume\ Eff_2 = \frac{q_{fin,2}}{V_{box}} = 0.107\,\frac{w}{cm^3}$$

Mass Efficiency

$$Mass\ Eff_1 = \frac{q_{fin,1}}{m_f} = 0.029\frac{w}{g}$$

$$Mass\ Eff_2 = \frac{q_{fin,2}}{m_f} = 0.050\frac{w}{g}$$

C.4 Cone Fin

Given Information

$$T_{b,1} = 63.21\ °C$$

$$T_{b,2} = 60.54\ °C$$

$$T_{t,1} = 50.86\ °C$$

$$T_{t,2} = 49.05\ °C$$

$$T_{sur} = 24.4\ °C$$

$$P_{wattmeter,1} = 32.2\ W$$

$$P_{wattmeter,1} = 31.4\ W$$

$$P_{loss} = 3\ W$$

$$L_c = 0.0254\ m$$

$$A_t = 0.00739\ m^2$$

$$A_c = 0.00025\ m^2$$

$$P = 0.108\ m^2$$

$$k_{air} = 14.9\ \frac{w}{mK}$$

$$m_f = 0.0232\ kg$$

$$V_{box} = 3.278 * 10^{-5} m^3$$

$$\theta_{b,1} = T_b - T_{sur} = 38.81\ °C$$

$$\theta_{b,2} = T_b - T_{sur} = 36.14\ °C$$

Resistance

$$R_{total,1} = \frac{T_{b,1} - T_{sur}}{P_{wattmeter} - P_{loss}} = 1.37\frac{K}{W}$$

$$R_{total,2} = \frac{T_{b,2} - T_{sur}}{P_{wattmeter} - P_{loss}} = 1.27\frac{K}{W}$$

Total Coefficient of Heat Transfer

$$h_{total,1} = \frac{1}{R_{total,1}A_t} = 99.04\frac{w}{m^2k}$$

$$h_{total,2} = \frac{1}{R_{total,2}A_t} = 106.35\frac{w}{m^2k}$$

Radiative Heat Transfer Coefficient

$$h_{r,1} = \sigma(T_s^2 + T_{sur}^2)(T_s + T_{sur}) = 0.0106\frac{w}{m^2k}$$

$$h_{r,2} = \sigma(T_s^2 + T_{sur}^2)(T_s + T_{sur}) = 0.0105\frac{w}{m^2k}$$

Fin Coefficient of Heat Transfer

$$\frac{\theta_L}{\theta_{b,1}} = \frac{1}{I_0(mr_b)K_1(mr_L) + I_1(mr_L)K_0(mr_b)} \rightarrow h_{fin,1} = 85.94\frac{w}{m^2k}$$

$$\frac{\theta_L}{\theta_{b,2}} = \frac{1}{I_0(mr_b)K_1(mr_L) + I_1(mr_L)K_0(mr_b)} \rightarrow h_{fin,1} = 85.94\frac{w}{m^2k}$$

Fin parameter

$$m_1 = \sqrt{\frac{h_{fin,1}P}{k_{air}A_C}} = 30.14\ m^{-1}$$

$$m_2 = \sqrt{\frac{h_{fin,2}P}{k_{air}A_C}} = 30.14\ m^{-1}$$

Modified Heat Transfer Rate

$$M_1 = \theta_{b,1}(h_{fin.1}Pk_{air}A_c)^{1/2} = 4.42\;W$$

$$M_2 = \theta_{b,2}(h_{fin,2}Pk_{air}A_c)^{1/2} = 4.41\;W$$

Fin Heat Transfer

$$q_{fin,1} = M_1\frac{\sinh(m_1L_c) + \frac{h_{fin.1}}{m_1k_{air}}\cosh(m_1L_c)}{\cosh(m_1L_c) + \frac{h_{fin.1}}{m_1k_{air}}\sinh(m_1L_c)} = 3.28\;W$$

$$q_{fin,2} = M_2\frac{\sinh(m_2L_c) + \frac{h_{fin.2}}{m_2k_{air}}\cosh(m_2L_c)}{\cosh(m_2L_c) + \frac{h_{fin.2}}{m_2k_{air}}\sinh(m_2L_c)} = 3.06\;W$$

Fin Efficiency

$$\eta_{fin,1} = \frac{\tanh\,(m_1L_c)}{m_1L_c} = 0.91$$

$$\eta_{fin,2} = \frac{\tanh\,(m_2L_c)}{m_2L_c} = 0.91$$

Fin Effectiveness

$$\epsilon_1 = \frac{q_{fin,1}}{\theta_{b,1}h_{fin,2}A_c} = 1.94$$

$$\epsilon_2 = \frac{q_{fin,2}}{\theta_{b,2}h_{fin,2}A_c} = 1.94$$

Volume Efficiency

$$Volume\;Eff_1 = \frac{q_{fin,1}}{V_{box}} = 0.100\,\frac{w}{cm^3}$$

$$Volume\;Eff_2 = \frac{q_{fin,2}}{V_{box}} = 0.093\,\frac{w}{cm^3}$$

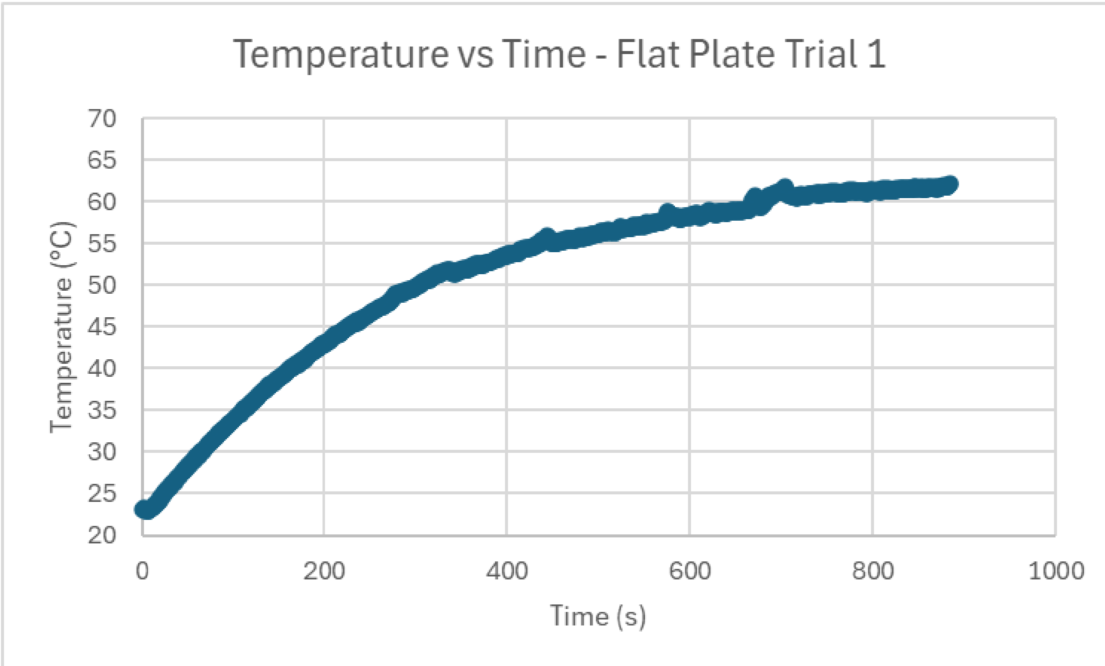
Mass Efficiency

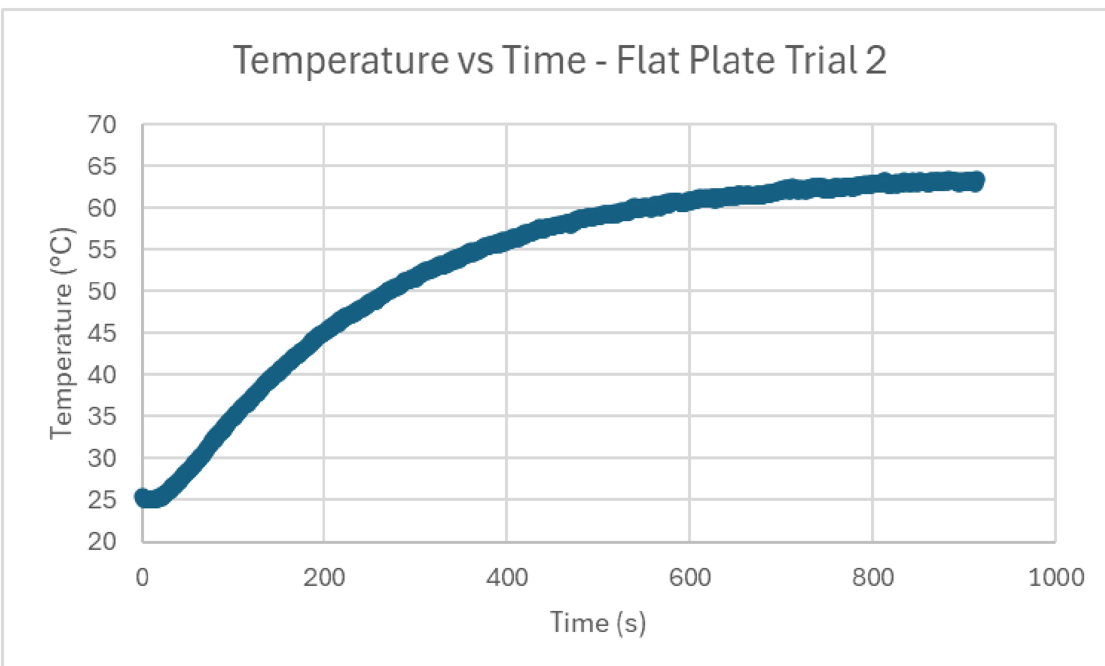
$$Mass\ Eff_1 = \frac{q_{fin,1}}{m_f} = 0.142\,\frac{w}{g}$$

$$Mass\ Eff_2 = \frac{q_{fin,2}}{m_f} = 0.132\,\frac{w}{g}$$

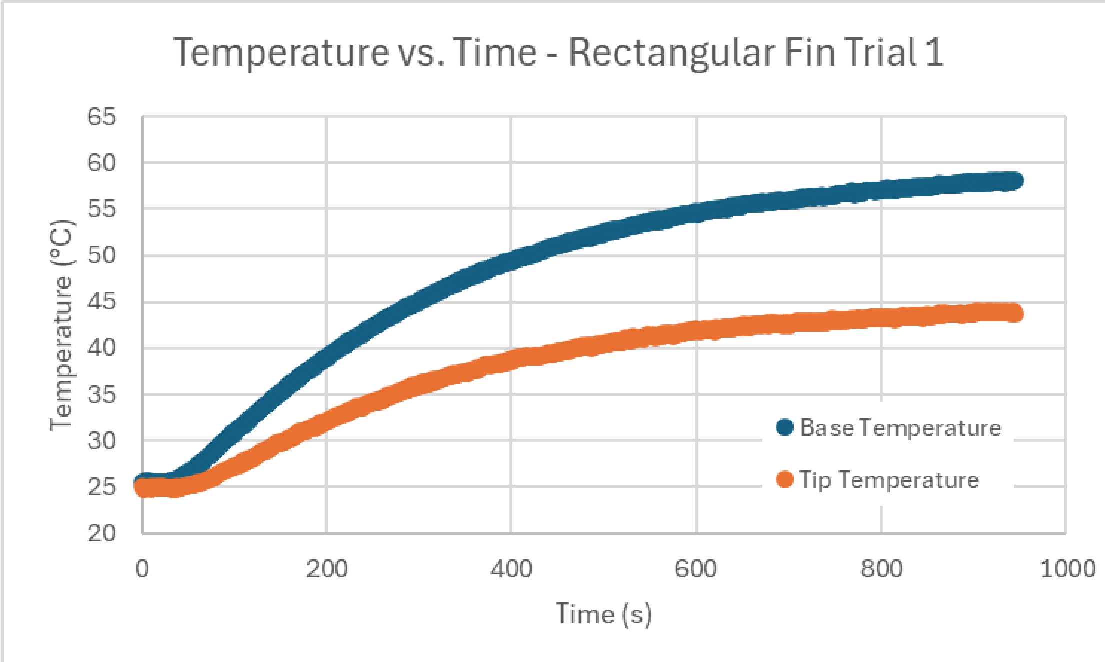
Appendix D Raw Data for Each Experiment

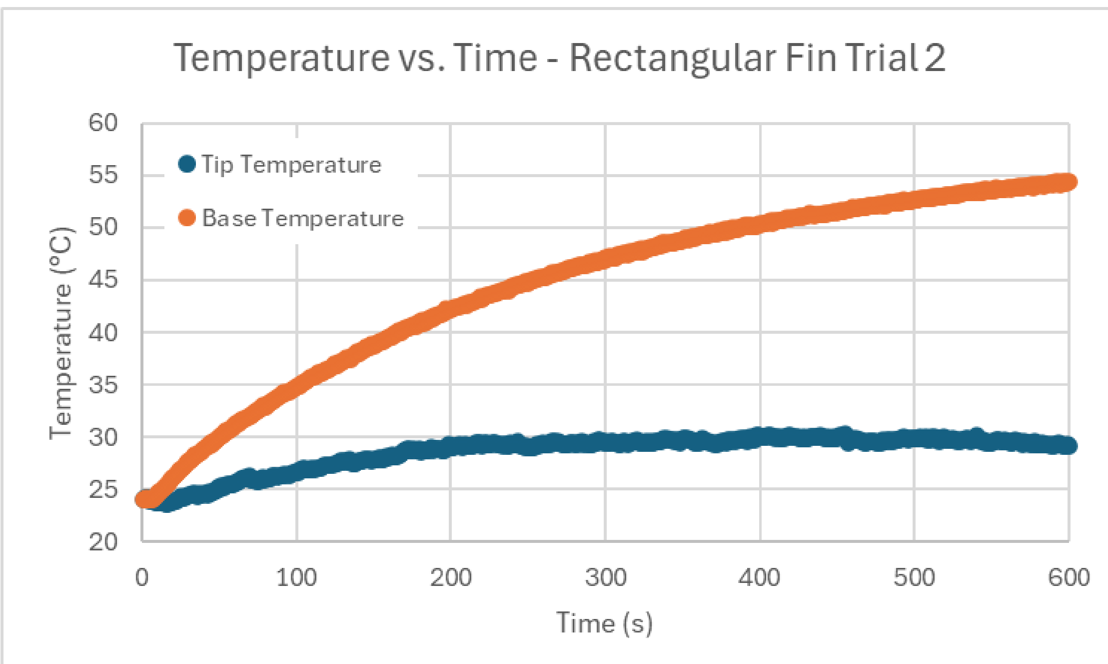
D.1 Flat Plate



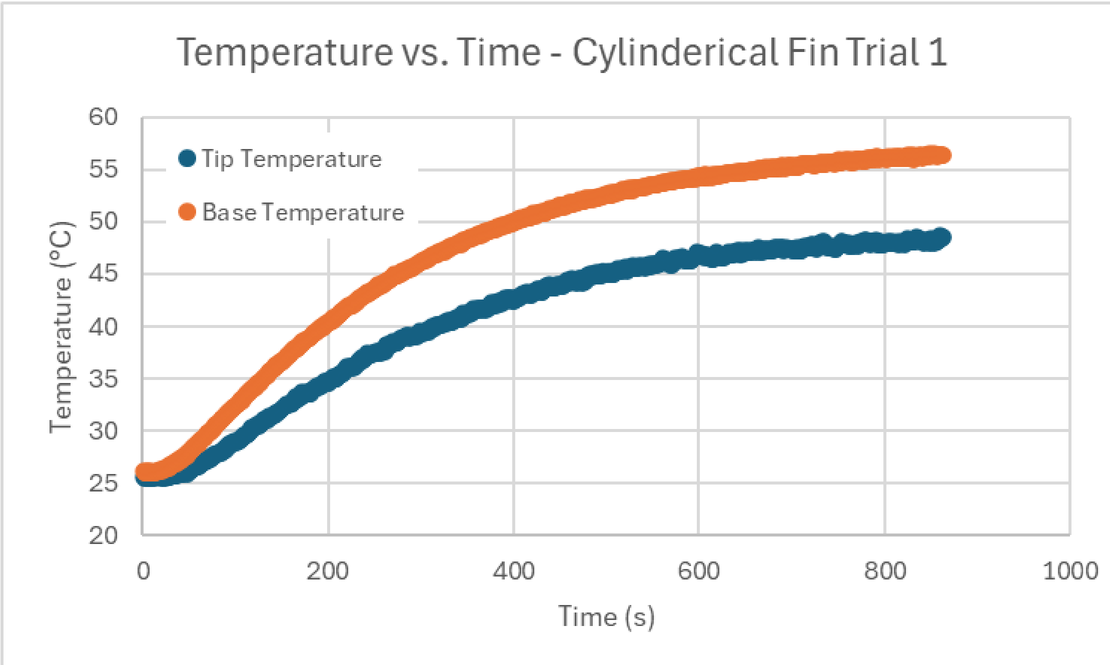


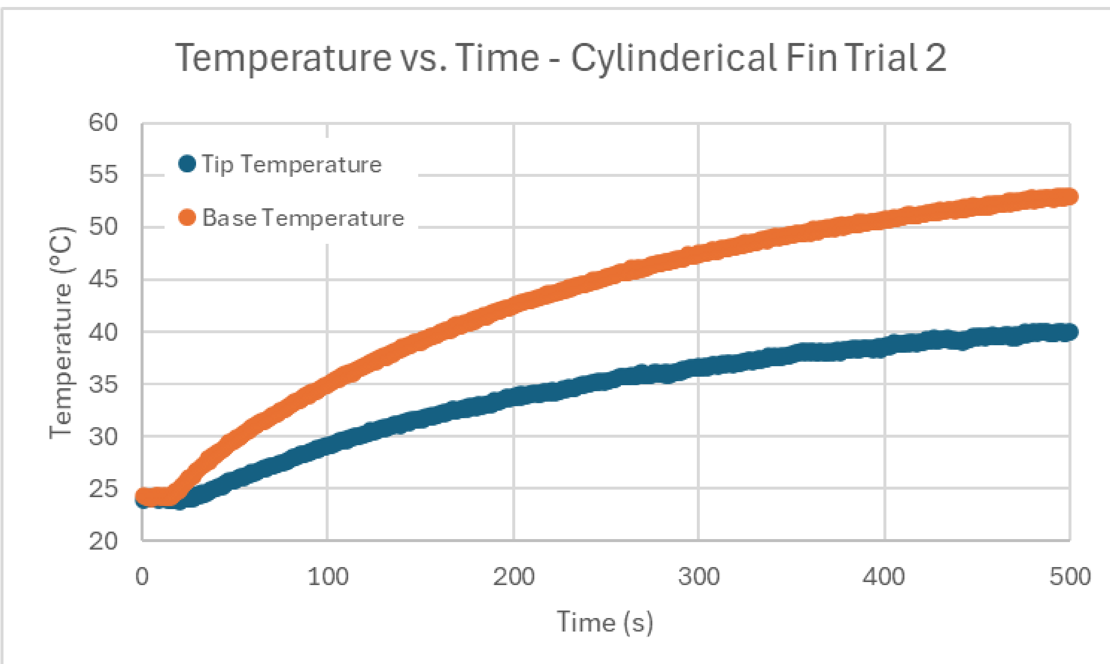
D.2 Rectangular Fin



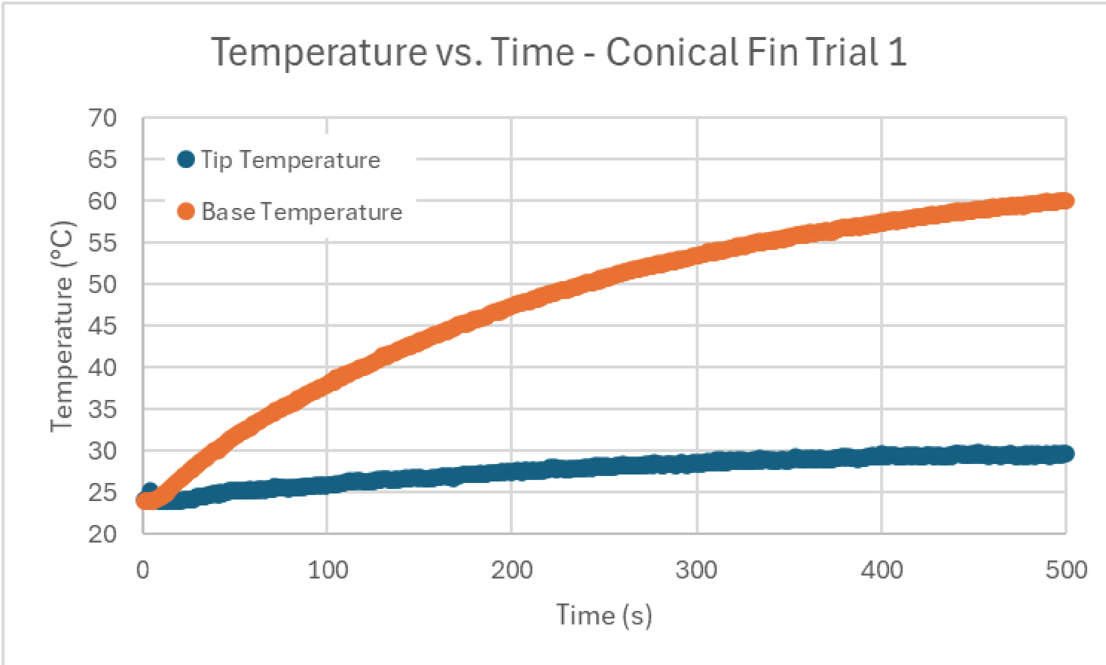


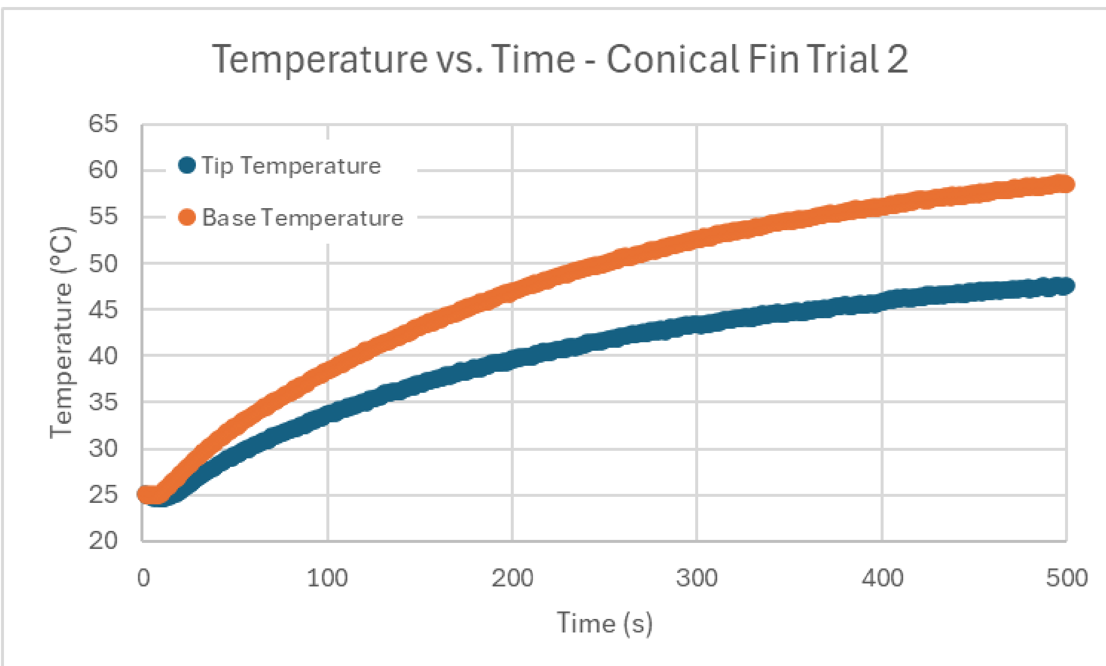
D.3 Cylindrical Fin





D.4 Cone Fin


Temperature vs. Time - Conical Fin Trial 1
Tip Temperature
Base Temperature
Temperature (°C)
70
65
60
55
50
45
40
35
30
25
20
0
100
200
300
400
500
Time (s)


Temperature vs. Time - Conical Fin Trial 2
Tip Temperature
Base Temperature
Temperature (°C)
65
60
55
50
45
40
35
30
25
20
0
100
200
300
400
500
Time (s)